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# AN EMPIRICAL INVESTIGATION OF THE EFFECT OF HETEROSCEDASTICITY AND HETEROGENEITY OF VARIANCE ON THE ANALYSIS OF COVARIANCE AND THE JOHNSON-NEYMAN TECHNIQUE

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U. S. Army

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The robustness of the Johnson-Neyman technique and analysis of covariance (ANCOVA) to violations of assumptions of homoscedasticity and homogeneity of variance was tested through the use of Monte Carlo computer procedures. The study simulated a one-way, fixed-effects analysis with two treatment groups, one criterion, and one covariate. Five fixed values of the covariate were selected with zero mean and unit variance, while the values of Y were varied randomly with a constant regression coefficient of .75. Four combinations — over

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20. ABSTRACT (Continued)

of group sizes (10,10;10,20;20,10;20,20), five combinations of group variances (1,1;1,2;2,1;1,5;5,1), and five forms of heteroscedasticity (combined in 18 different pairs), were studied. These conditions were combined to produce 186 different simulated experimental conditions. For each simulated condition 3000 pseudo-random samples were generated and sampling distributions relevant to the Johnson-Neyman technique and ANCOVA were compiled.

Results indicated that ANCOVA is robust to violations of assumptions of homoscedasticity and homogeneity of variance, both singly and in combination, when group sizes were equal. For cases of different group sizes and heterogeneous variances a predictable bias was observed. When the larger variance was combined with the smaller group size the bias was conservative. When the pairings were reversed the bias was nonconservative. The Johnson-Neyman technique was sensitive to violation of the assumption of homoscedasticity for both equal and unequal group sizes. The effect of heteroscedasticity was to order the probability that any fixed value of X would be included in a region of significance in a sequence parallel to the form of heteroscedasticity. That is, in general, as the variance for a fixed value of X increased, the probability of including that value of the covariate in a region of significance increased.




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# AN EMPIRICAL INVESTIGATION OF THE EFFECT OF HETEROSCEDASTICITY AND HETEROGENEITY OF VARIANCE ON THE ANALYSIS OF COVARIANCE AND THE JOHNSON-NEYMAN TECHNIQUE

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## FOREWORD

In the course of conducting research, the U.S. Army Research Institute for the Behavioral and Social Sciences (ARI) makes use of many statistical models and techniques suited to a wide variety of information gathering and hypothesis testing. The nature of conditions in the field, where the best Army data can be collected, often results in violations of assumptions assumed to be critical for the validity of specific statistical operations or models. The information provided in this Technical Paper is useful, not only in the Individual Training and Skill Evaluation Technical Area but to experimenters and analysts in other areas of behavior science research who need to determine the appropriateness of using Analysis of Covariance (ANCOVA) or the Johnson-Neyman technique.

The entire research is responsive to requirements of RDT&E Project PE62722A777, Individual Training Technology, FY 1978 Work Program, and to special requirements of the Deputy Chief of Staff for Personnel.

Jos. Zeidner

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AN EMPIRICAL INVESTIGATION OF THE EFFECT OF HETEROSCEDASTICITY AND HETEROGENEITY OF VARIANCE ON THE ANALYSIS OF COVARIANCE AND THE JOHNSON-NEYMAN TECHNIQUE.

BRIEF

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Requirement:

To determine the effects of violating the assumptions of homoscedasticity and homogeneity of variance on significance tests associated with ANCOVA and the Johnson-Neyman technique.

Procedure:

The robustness of the Johnson-Neyman technique and analysis of covariance (ANCOVA) to violations of assumptions of homoscedasticity and homogeneity of variance was tested through use of Monte Carlo computer procedures. The study simulated a one-way, fixed-effects analysis with two treatment groups, one criterion, Y, and one covariate, X. Five fixed values of the covariate were selected with zero mean and unit variance, while the values of Y were varied randomly with a constant regression coefficient of .75. Four combinations of group sizes (10,10;10,20;20,10;20,20), five combinations of group variances (1,1;1,2;2,1;1,5;5,1), and five forms of heteroscedasticity (combined in 18 different pairs), were studied. These conditions were combined to produce 186 different simulated experimental conditions. For each simulated condition, 3000 pseudo-random samples were generated and sampling distributions relevant to the Johnson-Neyman technique and ANCOVA were compiled.

Findings:

Results indicated that ANCOVA is robust to violations of assumptions of homoscedasticity and homogeneity of variance, both singly and in combination, when group sizes were equal. For cases of different group sizes and heterogeneous variances a predictable bias was observed. When the larger variance was combined with the larger group size the bias was conservative. When the pairings were reversed the bias was non-conservative. The Johnson-Neyman technique was sensitive to violation of the assumption homoscedasticity for both equal and unequal group sizes. The effect of heteroscedasticity was to order the probability that any fixed value of X would be included in a region of significance in a sequence parallel to the form of heteroscedasticity. That is, in general, as the variance for a fixed value of X increased, the probability of including that value of the covariate in a region of significance increased. As observed with ANCOVA, the Johnson-Neyman technique was robust to heterogeneity of variance when group sizes were equal. However, when group sizes were

not equal the empirical probabilities were biased in a non-conservative direction when the larger variance was combined with the smaller group size, and in a conservative direction when the larger variance and larger group size were combined.

Utilization of Findings:

In many empirical situations such as practical field experiments conducted within the Army it is not possible to meet all the assumptions of statistical models. If one or more of the assumptions is violated, the user has the choice of abandoning the model or proceeding with the analysis at some risk. The results of this study may be used by the investigator to estimate the degree of bias in the test of significance associated with either ANCOVA or the Johnson-Neyman technique.

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The effect on Type I error rates and power of violations of assumptions of mathematical models underlying statistical analyses has been studied for some time. Although many of the theoretical consequences have been derived, it was only with the advent of high speed computers that the empirical consequences could be determined with facility. As pointed out in a recent review by Glass et al. (1972), "the assumptions of most mathematical models are always false to a greater or lesser degree." The purpose of this study was to compare the empirical results of certain reasonable violations of two of the assumptions underlying the mathematical models associated with the analysis of covariance (Fisher, 1932) and the Johnson-Neyman technique (Johnson and Neyman, 1936).

Although both the analysis of covariance (ANCOVA) and the Johnson-Neyman technique incorporate regression methods in order to increase the precision of an experimental design, ANCOVA assumes homogeneity of regression whereas the Johnson-Neyman technique is not based on this assumption. Furthermore, there are two current areas of great interest in educational psychology research where heterogeneity of regression might be expected (e.g., Aptitude Treatment Interaction (Bracht, 1970) and Moderator Variables (Bartlett et al., 1969; Ghiselli, 1963)). Therefore, the Johnson-Neyman technique becomes an important alternative to ANCOVA.

The Johnson-Neyman technique defines a region along the covariate where significant treatment differences exist between two groups. Unlike ANCOVA, the Johnson-Neyman technique does not test a hypothesis, but yields a confidence interval. The region not included in the confidence interval, or the region of significance, may be a single continuous region or two distinct regions where one group is significantly better on the criterion in one region, and significantly inferior in the other region.

The mathematical model for both ANCOVA and the Johnson-Neyman technique is:

$$Y_{kj} = \alpha_j + \beta_j X_{kj} + e_{kj}$$

where

$\alpha_j$ = Y intercept of the Y-on-X regression line for group j.

$\beta_j$ = regression slope of the Y-on-X regression line for group j.

$e_{kj}$ = error of estimate for score k in group j.

The $e_{kj}$ are assumed to be independently and normally distributed with 0 mean and homogeneous variance, $\sigma^2 = \sigma^2_{y|x_{ij}}$. That is, it is assumed that errors of estimate are homogeneous for each fixed value of the concomitant variable, $X_j$, both within a treatment group (homoscedasticity), and between treatment groups (homogeneity of variance).

In addition to the assumption of constant error variance, both models assume: a linear relationship between the criterion and concomitant variable and that values of the concomitant variable are fixed and measured without error. ANCOVA makes the additional assumption of homogeneity of regression, i.e., $\beta_j = \beta_w$ for all groups.

In many empirical situations it is not possible to meet all the assumptions of statistical models. If one or more of the assumptions is violated, the user has the choice of abandoning the model or proceeding with the analysis at some risk. Theoretical discussions of a technique often enable the investigator to determine whether his test is biased in a conservative or liberal direction, while empirical investigations of violations often enable the investigator to estimate the degree of bias.

The purpose of this study was to compare, by Monte Carlo methods, the effects of violating assumptions of homoscedasticity and homogeneity of variance on significance tests associated with ANCOVA and the Johnson-Neyman technique. The research questions asked are:

1. Are the Johnson-Neyman technique and ANCOVA robust to the violation of the assumption of homoscedasticity?

2. Are the Johnson-Neyman technique and ANCOVA robust to the violation of the assumption of homogeneity of variance?

3. Are the Johnson-Neyman technique and ANCOVA robust to the simultaneous violation of the assumptions of homoscedasticity and homogeneity of variance?

REVIEW OF RELATED LITERATURE

ANCOVA is an extension of analysis of variance (ANOVA) and regression analysis. As such, assumptions underlying ANCOVA include all those associated with ANOVA and regression analysis as well as the assumption of homogeneity of regression. Therefore, results from studies of the robustness of ANOVA to violations of some of its assumptions might be expected to extend to ANCOVA. Particularly pertinent to the current study is research on violation of the assumption of homogeneity of variance. The effect of heterogeneous variances on Type I error rates has been investigated theoretically (Box, 1954; Scheffé, 1959) and empirically (Norton, as reported in Lindquist, 1953).

Scheffé (1959) reports the work of Hsu who calculated the exact probability of Type I error rates for a two-tailed _t_ test at the .05 level for three different pairs of sample sizes (15,5;5,3 and 7,7) and 9 ratios of population variances $\sigma_1^2/\sigma_2^2$ (0,.1,.2,.5,1,2,5,10,$\infty$). Scheffé (1959) extended Hsu's data to large samples in the two-group case and studied 4 ratios of sample sizes (1,2,5,$\infty$) and 7 ratios of population variances, $\sigma_1^2/\sigma_2^2$ (0,.2,.5,1,2,5,$\infty$).

Scheffé concluded that inequality of variances has little effect on Type I error rate when sample sizes are equal, but has serious effects when sample sizes are not equal. In general, when the larger variance is associated with the smaller sample size the true probability of Type I error was found to exceed the nominal value, and when the larger variance and sample size were paired the Type I error rate was found to be less than the nominal value.

Box (1954) calculated exact probabilities of Type I errors for fixed-effect ANOVA F tests at a nominal 5 percent level. The results were obtained for 3, 5, and 7 groups, variance ratios of 1:2:3; 1:1:3; 1:1:1:1:3; and 1:1:1:1:1:1:7, and for 11 combinations of equal and unequal group sizes. In general, it was observed that the F test is robust to the violation of homogeneity of variance for equal sample sizes, but not for unequal sample sizes. When the largest variance was associated with the smallest sample size, the actual Type I error rate was greater than the nominal value; when the largest variance was associated with the largest sample size, the value of $\alpha$ was smaller than nominal.

The robustness of the F test to heterogeneous variances in balanced designs reported by Scheffé (1959) and Box (1954) was consistent with an empirical investigation by Norton (as reported in Lindquist, 1953). Using a Monte Carlo simulation technique, he constructed (by hand) populations of 10,000 cases each and sampled from these populations in order to create empirical sampling distributions. In the phase of his study in which he investigated heterogeneity of variance, populations were normal with equal means, but different variances

($\sigma_x^2$ = 25, 100, and 225). Marked heterogeneity of variance (1:4:9) resulted in a small but predictable bias in the Type I errors and empirical values were generally greater than nominal values.

Little research on the effect of heterogeneity of variance on robustness of ANCOVA has been conducted. A theoretical paper by Potthoff (1965) showed that the sensitivity of ANCOVA to heterogeneous variances depended on the ratio $n_1\sigma_{x_1}/n_2\sigma_{x_2}$ where n is sample size and $\sigma_x$ is the standard error of the covariate. Three empirical studies have been conducted concerning effects of violating the assumption of homogeneity of variance while simultaneously violating the assumption of homogeneity of regression (Peckham, 1968; Hamilton, 1972; and McClaren, 1972). In all studies, variance of the criterion measure ($\sigma_y^2$) and variance of the covariate ($\sigma_x^2$) were held constant while varying population regression slopes ($\beta$), thereby producing a concomitant change in $\sigma_{y|x}^2$ (where $\sigma_{y|x}^2 = \sigma_y^2 - \sigma_x^2\beta^2$). In the model for ANCOVA the assumption of homogeneity of variance applies to the variance error of estimate ($\sigma_{y|x}^2$).

Peckham (1968) varied regression slopes (and $\sigma_{y|x}^2$), number of groups and sample size, although sample size was equal between groups for all of his comparisons. Values of the covariate were fixed and were chosen to conform as closely as possible to a normal distribution. Peckham found that there were very small discrepancies in the actual and empirical significance levels. In general, he observed that as the degree of heterogeneity of the regression slopes (and heterogeneity of variance) increased, the empirical rate of Type I errors was less than the nominal value.

In McClaren's (1972) study, the number of treatment groups were 2, 3, and 5, and the sample sizes were 20, 30, 40, 100, or 200, and regression slopes were .1,.2,.3,.4,.5,.6,.7,.8, and .9. The average slope across treatments was held constant at .5 for 180 out of the 183 simulated conditions. The values of the concomitant variable were fixed, with zero mean and unit variance. For equal group sizes, he found that as degree of heterogeneity of regression increased (and heterogeneity of variance increased), the empirical level of significance became more conservative. For unequal sample sizes, the results parallel the effect of violating the assumption of homogeneity of variance reported by Box (1954) and Scheffé (1959). That is, when the smallest regression coefficient and the largest variance were combined with the smallest sample size, the empirical significance levels were biased in a non-conservative direction and when the pairings were reversed the test was conservative.

Hamilton (1972) restricted the number of groups to two and varied sizes, with distributions of the criterion and the covariate being bivariate normal. Hamilton's results, in general, parallel the results of the effect of violating the assumption of homogeneity of variance

reported by Box (1954) and Scheffé (1959). Hamilton found ANCOVA robust to the violation of homogeneity of regression (and variance) for equal sample sizes, but observed large discrepancies in empirical and nominal alpha levels for unequal sample sizes. When the larger sample size occurred with the larger regression slope, and therefore smaller $\sigma^2_{y|x}$, the empirical alpha levels were greater than corresponding nominal alpha levels. When the smaller group size was paired with the larger regression slope (and smaller $\sigma^2_{y|x}$), he observed that empirical alpha levels were less than corresponding nominal levels. Hamilton's study appears to present evidence for the generalizability of results from the study of the effect of violating the assumptions associated with ANOVA, as suggested by Cochran (1957) and Winer (1971).

In one condition, Hamilton studied the same combination of equal sample sizes, number of groups, and regression coefficients as Peckham and as McClaren, but failed to replicate their results; a comparison is presented in Table 1. Whereas Hamilton's values were close to nominal alpha, Peckham and McClaren observed a conservative bias in empirical alpha levels where group sizes were equal and regression slopes heterogeneous. It is difficult to resolve the discrepancies in the results of these studies. Although it is impossible to determine simple effects of violating the assumption of homogeneity of regression or variance from the results of Hamilton (1972), McClaren (1972), and Peckham (1968), an analytical study by Atiquallah (1964) suggests that the F test of ANCOVA is robust to the violation of the assumption of homogeneity of regression when sample size is large and the means of the concomitant variable are equal; otherwise, the test is biased in a conservative direction.

There is no study examining the unique effect of heteroscedasticity on the robustness of ANCOVA. For an overview of ANCOVA and the effects of other violations on ANOVA and ANCOVA, comprehensive reviews by Glass et al. (1971) and Elashoff (1969) are available.

The Johnson-Neyman technique has not received as much attention as ANCOVA and little is known concerning the effects of violating assumptions underlying this statistical method. As originally presented, the Johnson-Neyman technique was designed for the case of two predictor variables, two treatment groups, and one criterion (Johnson and Neyman, 1936), and papers on the technique have concerned extension to cases of "n" predictor variables and "k" groups (Abelson, 1953; Potthoff, 1964).

Table 1

Comparison of Actual and Nominal Levels of Significance in Simulations of "True" Experiments Obtained by Peckham (1968), Hamilton (1972), and McClaren (1972)

| Regression Coefficients | Group Sizes $n_1=10, n_2=10$ Nominal Alpha .10 | .05 | .01 | $n_1=20, n_2=20$ Nominal Alpha .10 | .05 | .01 |
|---|---|---|---|---|---|---|
| **Peckham** | | | | | | |
| .5,.5 | .094 | .052 | .013 | .102 | .049 | .013 |
| .4,.6 | .096 | .050 | .010 | .089 | .045 | .009 |
| .3,.7 | .091 | .045 | .011 | .097 | .051 | .009 |
| .2,.8 | .076 | .039 | .006 | .076 | .038 | .008 |
| .1,.9 | .055 | .029 | .004 | .055 | .027 | .005 |
| **Hamilton** | | | | | | |
| .5,.5 | .098 | .049 | .013 | .115 | .057 | .015 |
| .4,.6 | .099 | .053 | .011 | .103 | .052 | .011 |
| .3,.7 | .104 | .051 | .010 | .100 | .054 | .012 |
| .2,.8 | .105 | .058 | .013 | .103 | .051 | .009 |
| .1,.9 | .109 | .057 | .015 | .103 | .056 | .015 |
| **McClaren** | | | | | | |
| .5,.5 | | | | .105 | .047 | .012 |
| .4,.6 | | | | .099 | .049 | .009 |
| .3,.7 | | | | .090 | .049 | .011 |
| .2,.8 | | | | .074 | .037 | .010 |
| .1,.9 | | | | .060 | .022 | .004 |

## PROCEDURE

The effect of violating the following assumptions on F tests associated with ANCOVA, and regions of significance associated with the Johnson-Neyman technique were investigated:

1. heteroscedasticity
2. heterogeneity of variance
3. heterogeneity of variance and heteroscedasticity

The study simulated a simple one-way, fixed-effects analysis with two treatment groups, one criterion, and one covariate. Five fixed values of X, the covariate, assumed to be measured without error, with zero mean and unit variance, were selected. Four combinations of group sizes were used: 10,10;20,20;10,20; and 20,10; with an equal number of cases at each fixed value of X. All assumptions underlying the two methods were met except those under study. The value of the regression coefficient was held constant at $\beta = .75$, and the expected values of $\hat{Y}$ ($\hat{Y} = \beta X + \alpha$) for both groups were -0.8485, -0.4243, 0.0000, +0.4243, +0.8485. Nominal significance levels of .01 to .99, increasing in steps of .01, were used for comparison with empirical $\alpha$ levels.

### Heterogeneity of Variance

Three values of heterogeneity of variance were studied; $\sigma^2_{y|x}$ was set at 3 values: 1, 2, and 5. As $\sigma^2_{y|x}$ changes, while $\beta$ and $\sigma^2_x$ are held constant, there is a concomitant change in $\rho$ and $\sigma^2_y$; resulting values are shown in Table 2. $\sigma^2_{y|x} = 1$ was paired with $\sigma^2_{y|x} = 1$ (homogeneity of variance), $\sigma^2_{y|x} = 2$, and $\sigma^2_{y|x} = 5$. These pairs combined with each of the pairs of group sizes produced 11 experimental conditions ($\sigma^2_{y|x} = 1$ paired with $\sigma^2_{y|x} = 1$ and group size combinations 10,20 and 20,10 are equivalent).

Table 2

Values of $\sigma^2_y$ and $\rho$ where

$\sigma^2_{y|x} = \sigma^2_y(1-\rho^2)$ and $\alpha^2_x = 1, \beta = .75$

| $\sigma^2_{y\|x}$ | $\sigma^2_y$ | $\rho$ |
|---|---|---|
| 1 | 1.56 | 0.60 |
| 2 | 2.56 | 0.47 |
| 5 | 5.56 | 0.32 |

## Heteroscedasticity

Five forms of heteroscedasticity were studied. The average value of $\sigma^2_{y|x}$ was held constant at 1, 2, or 5, while values of the error variance for each fixed X ($\sigma^2_{y|x_{ij}}$) were distributed over the 5 fixed values of X so that the following forms were approximated:

a. Equal $\sigma^2_{y|x_{ij}}$ for all $X_{ij}$, homoscedasticity (form A).

b. Greatest $\sigma^2_{y|x_{ij}}$ in the center with gradually decreasing $\sigma^2_{y|x_{ij}}$ to either end point (form B).

c. Greatest $\sigma^2_{y|x_{ij}}$ at the largest value of X and gradually decreasing to the smallest value of X (form C).

d. Greatest $\sigma^2_{y|x_{ij}}$ at the smallest value of X and gradually increasing to the largest value of X (form D).

e. Smallest $\sigma^2_{y|x_{ij}}$ in the center gradually increasing to either end point (form E).

Table 3 gives $\sigma^2_{y|x_{ij}}$ at each fixed point of X for the 5 forms where $\sigma^2_{y|x} = 1$, $\sigma^2_{y|x} = 2$, and $\sigma^2_{y|x} = 5$, and a graphic representation of the forms of heteroscedasticity is presented in Figure 1. When combined in pairs, the 5 forms of heteroscedasticity produce 25 combinations (including the null case). The pairings of form D with forms A, B, C, and E are the reverse repetitions of pairs CA, CB, CD, and CE and, hence, these were not investigated. Thus, there were 18 combinations of heteroscedasticity which were combined with 11 combinations of group sizes and variance ratios. Where group sizes and variances were equal, mirror images of combinations were not investigated. Therefore, the total number of simulated experimental conditions was reduced to 186.

Table 3

Values for $\sigma^2_{y|x_{ij}}$ where $\sigma^2_{y|x}$=1,2, or 5, for the Five Forms of Heteroscedasticity for the Five Fixed Values of the Concomitant Variable, X

| $X_{ij}$ Fixed Value of X | Form of Heteroscedasticity A | B | C | D | E |
|---|---|---|---|---|---|
| | $\sigma^2_{y\|x} = 1$ | | | | |
| -1.4142 | 1.00 | 0.45 | 0.35 | 1.75 | 1.50 |
| -0.7071 | 1.00 | 0.925 | 0.70 | 1.20 | 0.85 |
| 0.0000 | 1.00 | 2.25 | 1.00 | 1.00 | 0.30 |
| +0.7071 | 1.00 | 0.925 | 1.20 | 0.70 | 0.85 |
| +1.4142 | 1.00 | 0.45 | 1.75 | 0.35 | 1.50 |
| | $\sigma^2_{y\|x} = 2$ | | | | |
| -1.4142 | 2.00 | 0.90 | 0.70 | 3.50 | 3.00 |
| -0.7071 | 2.00 | 1.85 | 1.40 | 2.40 | 1.70 |
| 0.0000 | 2.00 | 4.50 | 2.00 | 2.00 | 0.60 |
| +0.7071 | 2.00 | 1.85 | 2.40 | 1.40 | 1.70 |
| +1.4142 | 2.00 | 0.90 | 3.50 | 0.70 | 3.00 |
| | $\sigma^2_{y\|x} = 5$ | | | | |
| -1.4142 | 5.00 | 2.25 | 1.75 | 8.75 | 7.50 |
| -0.7071 | 5.00 | 4.625 | 3.50 | 6.00 | 4.25 |
| 0.0000 | 5.00 | 11.25 | 5.00 | 5.00 | 1.50 |
| +0.7071 | 5.00 | 4.625 | 6.00 | 3.50 | 4.25 |
| +1.4142 | 5.00 | 2.25 | 8.75 | 1.75 | 7.50 |

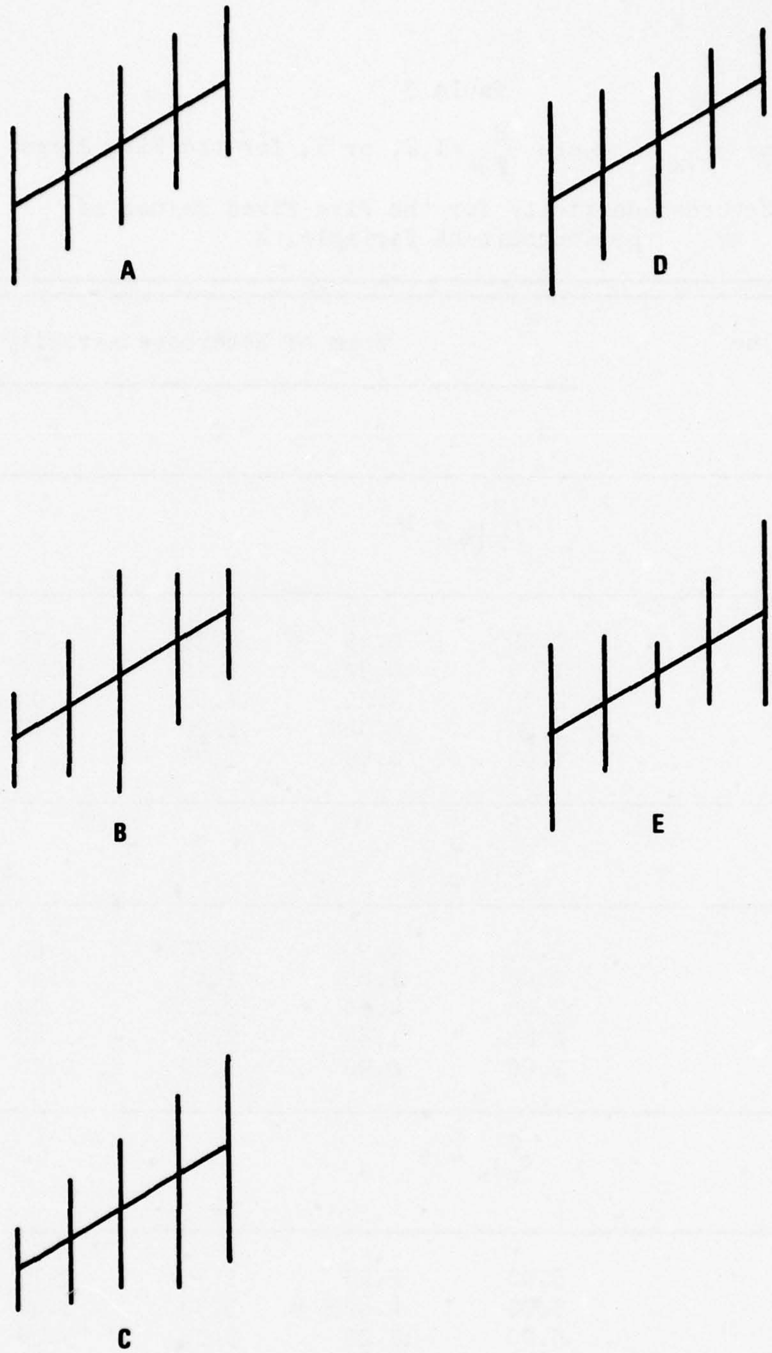


Fig. 1. Forms of heteroscedasticity studied (form A is homoscedastic). Each vertical line is in standard deviation units, and the average unit is 1.

## The Simulation Procedure

For each of the 186 simulated conditions, the following parameters were set:

1. The fixed set of values for X, the concomitant variable.

2. The number of observations per fixed X for each group.

3. The expectation of the dependent variable $Y_i$ for each fixed $X_{ij}$.

4. The standard deviation, $\sigma_{y|x_{ij}}$, of each fixed $X_{ij}$ for each group.

5. The number of samples, which was set equal to 3000.

UNIVAC 1108 Math-Pak (1970) subroutines RANDN and RANDU were used to generate random numbers. RANDU computes uniformly distributed pseudo-random real numbers between 0 and 1, whereas, RANDN produces sets of pseudo-random numbers which are normally distributed with a specified mean and standard deviation. Studies (UNIVAC 1108 Math-Pak Programmer's Reference Manual, UP-7542, section 14.3) have shown that the initialization number for the random number generator is critical for ensuring properties of randomness. Therefore, RANDU was used to supply a new starting number to RANDN for each of the 3000 samples in each run. A check on randomness of the generated sequences was made for each of the 186 initialization numbers. Using each of the initialization numbers an empirical F distribution was created for the case where no assumptions were violated and group sizes were each equal to 10. The Kolmogorov-Smirnov test was used to assess the goodness-of-fit of these empirical F distributions to theoretical F distributions; at $\alpha = .05$, the test failed to reject the hyopthesis of no difference between the nominal and empirical distributions for all initialization numbers used. In addition, checks of the randomness and normality of the samples generated by RANDN have been run and satisfactory results have been reported by Hamilton (1972).

## Goodness-of-fit Procedure

A total of 186 different goodness-of-fit testing situations were simulated in this study. Both ANCOVA and the Johnson-Neyman technique were carried out for each experimental condition. Ninety-nine F values were computed for each simulation at significance levels ranging from .01 to .99 in steps of .01. The goodness-of-fit of the empirical F distributions for ANCOVA to the theoretical F distributions was tested using the Kolmogorov-Smirnov one-sample goodness-of-fit test at $\alpha = .05$ (Siegel, 1956). In addition, the number of Type I errors was computed at each of the significance levels.

For the Johnson-Neyman technique, the probability that each fixed X would be included in a region of significance, as well as the total probability of obtaining any region of significance, and probabilities of obtaining central versus tail regions at nominal significance levels of .01, .025, .05, and .10 were computed.

## RESULTS

### The Analysis of Covariance

The results of the goodness-of-fit tests in each of the 186 simulated conditions are presented in Table 4. The symbol NS means that the Kolmogorov-Smirnov test failed to reject the null hypothesis that there was no difference between the empirical and nominal F distributions; the symbol S means that the Kolmogorov-Smirnov test rejected the null hypothesis. The forms of heteroscedasticity are represented by letters A,B,C,D, and E (see Figure 1, or Table 3). The empirical significance levels corresponding to the nominal levels of .10,.05,.02, and .01 for all experimental combinations are shown in Table 5.

### The Johnson-Neyman Technique

The empirical probability of the inclusion of each fixed value of X in a region of significance at nominal alpha levels of .10,.05,.025, and .01 under each of the 186 simulated experimental conditions is reported in Table 6. Table 7 shows empirical probabilities of obtaining any region of significance, when the nominal alpha level was set at .10,.05,.025, and .01.

Table 4

Results of the Kolmogorov-Smirnov Goodness-of-fit Tests of the Empirical to the Theoretical F Distribution for Each of the Simulated Experimental Conditions

| Group Size | | Variance | | Heteroscedasticity Combinations | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| G1[a] | G2[a] | G1[a] | G2[a] | AA | BB | CC | DD | EE | AB | BA | AC | CA | AE | EA | BC | CB | BE | EB | CD | CE | EC |
| 10 | 10 | 1 | 1 | NS | NS | NS | NS | NS | NS | -- | NS | -- | NS | -- | NS | -- | NS | -- | NS | NS | -- |
| | | 2 | 1 | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS |
| | | 5 | 1 | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS |
| 10 | 20 | 1 | 1 | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS |
| | | 2 | 1 | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S |
| | | 5 | 1 | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S |
| | | 1 | 2 | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S |
| | | 1 | 5 | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 20 | 20 | 1 | 1 | NS | NS | NS | NS | NS | NS | -- | NS | -- | NS | -- | NS | -- | NS | -- | NS | NS | -- |
| | | 2 | 1 | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS |
| | | 5 | 1 | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS | NS |

[a]G refers to group.

Table 5

Empirical Alpha Levels Corresponding to Nominal Alpha Levels of .10, .05, .02, and .01 for Each Simulated Experimental Condition

VARIANCE ERROR OF ESTIMATE GROUP ONE = 1  GROUP TWO = 1

PAIRS OF GROUP SIZES

| | G1 = 10, G2 = 10 | | | | G1 = 10, G2 = 20 | | | | G1 = 20, G2 = 10 | | | | G1 = 20, G2 = 20 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ALPHA = | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 |
| AA | .102 | .056 | .021 | .012 | .105 | .056 | .025 | .014 | | | | | .098 | .040 | .017 | .008 |
| BB | .096 | .044 | .019 | .008 | .102 | .048 | .016 | .007 | | | | | .101 | .047 | .019 | .008 |
| CC | .098 | .046 | .017 | .007 | .096 | .047 | .016 | .008 | | | | | .097 | .047 | .018 | .008 |
| DD | .103 | .048 | .017 | .007 | .099 | .047 | .017 | .009 | | | | | .106 | .043 | .021 | .011 |
| EE | .091 | .046 | .019 | .010 | .094 | .047 | .018 | .009 | | | | | .100 | .046 | .020 | .009 |
| AB | .090 | .042 | .016 | .008 | .108 | .057 | .024 | .013 | | | | | .108 | .057 | .023 | .011 |
| BA | | | | | .090 | .040 | .017 | .007 | | | | | | | | |
| AC | .102 | .052 | .023 | .011 | .102 | .047 | .018 | .010 | | | | | .111 | .057 | .025 | .012 |
| CA | | | | | .092 | .048 | .019 | .009 | | | | | | | | |
| AE | .102 | .050 | .020 | .009 | .092 | .047 | .018 | .008 | | | | | .102 | .047 | .018 | .009 |
| EA | | | | | .101 | .051 | .019 | .011 | | | | | | | | |
| BC | .097 | .052 | .017 | .008 | .091 | .042 | .013 | .006 | | | | | .089 | .044 | .018 | .006 |
| CB | | | | | .098 | .053 | .015 | .007 | | | | | | | | |
| BE | .092 | .040 | .014 | .007 | .098 | .051 | .020 | .009 | | | | | .094 | .045 | .021 | .010 |
| EB | | | | | .089 | .042 | .014 | .007 | | | | | | | | |
| CD | .102 | .051 | .020 | .010 | .095 | .051 | .021 | .010 | | | | | .108 | .055 | .021 | .010 |
| CE | .102 | .053 | .019 | .010 | .103 | .053 | .022 | .013 | | | | | .102 | .048 | .018 | .009 |
| EC | | | | | .100 | .047 | .020 | .008 | | | | | | | | |

Note: Where line is blank the experimental condition was not run due to its similarity to a condition which is reported in the table.

Table 5--Continued

VARIANCE ERROR OF ESTIMATE GROUP ONE = 2 GROUP TWO = 1

PAIRS OF GROUP SIZES

| | G1 = 10, G2 = 10 | | | | G1 = 10, G2 = 20 | | | | G1 = 20, G2 = 10 | | | | G1 = 20, G2 = 20 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ALPHA = | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 |
| AA | .100 | .047 | .020 | .011 | .145 | .082 | .034 | .018 | .070 | .037 | .012 | .005 | .109 | .055 | .020 | .011 |
| BB | .098 | .051 | .020 | .010 | .143 | .081 | .032 | .018 | .070 | .028 | .009 | .003 | .098 | .048 | .022 | .011 |
| CC | .093 | .047 | .018 | .008 | .141 | .080 | .038 | .019 | .064 | .022 | .008 | .002 | .098 | .050 | .021 | .009 |
| DD | .105 | .054 | .023 | .012 | .147 | .077 | .034 | .018 | .073 | .032 | .015 | .005 | .100 | .051 | .020 | .011 |
| EE | .110 | .057 | .020 | .009 | .151 | .078 | .039 | .021 | .067 | .029 | .009 | .004 | .101 | .049 | .020 | .010 |
| AB | .107 | .050 | .021 | .010 | .133 | .075 | .031 | .017 | .060 | .026 | .010 | .006 | .095 | .048 | .021 | .011 |
| BA | .100 | .046 | .016 | .006 | .135 | .076 | .036 | .019 | .065 | .029 | .010 | .004 | .091 | .043 | .016 | .009 |
| AC | .099 | .050 | .019 | .009 | .141 | .080 | .038 | .022 | .068 | .028 | .009 | .003 | .106 | .058 | .025 | .012 |
| CA | .103 | .049 | .021 | .011 | .139 | .081 | .033 | .019 | .072 | .031 | .012 | .006 | .098 | .048 | .018 | .006 |
| AE | .101 | .051 | .021 | .010 | .138 | .081 | .033 | .018 | .073 | .031 | .012 | .005 | .102 | .047 | .017 | .010 |
| EA | .094 | .046 | .019 | .009 | .141 | .078 | .035 | .020 | .072 | .030 | .010 | .004 | .107 | .056 | .020 | .010 |
| BC | .107 | .053 | .022 | .011 | .144 | .075 | .035 | .019 | .063 | .028 | .011 | .005 | .105 | .057 | .023 | .011 |
| CB | .100 | .048 | .016 | .007 | .148 | .082 | .038 | .018 | .071 | .033 | .012 | .005 | .100 | .051 | .020 | .010 |
| BE | .097 | .052 | .024 | .014 | .142 | .078 | .029 | .018 | .066 | .030 | .010 | .004 | .098 | .048 | .019 | .009 |
| EB | .108 | .055 | .019 | .007 | .147 | .084 | .035 | .020 | .065 | .028 | .006 | .002 | .091 | .045 | .019 | .007 |
| CD | .108 | .054 | .022 | .010 | .137 | .077 | .034 | .018 | .070 | .030 | .009 | .004 | .103 | .051 | .025 | .015 |
| CE | .096 | .046 | .018 | .010 | .141 | .081 | .037 | .020 | .066 | .028 | .007 | .002 | .106 | .053 | .025 | .012 |
| EC | .102 | .047 | .020 | .009 | .144 | .086 | .037 | .022 | .069 | .030 | .009 | .005 | .089 | .045 | .016 | .008 |

Table 5--Continued

VARIANCE ERROR OF ESTIMATE GROUP ONE = 5 GROUP TWO = 1

PAIRS OF GROUP SIZES

| | G1 = 10, G2 = 10 | | | | G1 = 10, G2 = 20 | | | | G1 = 20, G2 = 10 | | | | G1 = 20, G2 = 20 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ALPHA = | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 | .10 | .05 | .02 | .01 |
| AA | .105 | .054 | .021 | .012 | .197 | .119 | .064 | .043 | .044 | .015 | .003 | .002 | .096 | .051 | .019 | .008 |
| BB | .099 | .051 | .021 | .010 | .190 | .113 | .055 | .032 | .040 | .014 | .003 | .001 | .096 | .046 | .018 | .009 |
| CC | .096 | .051 | .022 | .012 | .187 | .117 | .060 | .037 | .040 | .011 | .003 | .001 | .107 | .056 | .022 | .009 |
| DD | .104 | .058 | .027 | .014 | .198 | .130 | .072 | .045 | .045 | .017 | .005 | .001 | .107 | .054 | .020 | .009 |
| EE | .103 | .052 | .022 | .008 | .206 | .132 | .069 | .040 | .041 | .012 | .004 | .001 | .103 | .052 | .022 | .012 |
| AB | .108 | .057 | .028 | .016 | .205 | .127 | .072 | .049 | .042 | .012 | .002 | .000 | .098 | .050 | .021 | .010 |
| BA | .095 | .044 | .016 | .008 | .192 | .118 | .063 | .040 | .047 | .019 | .005 | .003 | .097 | .049 | .023 | .009 |
| AC | .109 | .057 | .027 | .014 | .196 | .124 | .071 | .046 | .052 | .018 | .005 | .002 | .108 | .055 | .020 | .010 |
| CA | .104 | .056 | .021 | .010 | .202 | .125 | .073 | .047 | .040 | .016 | .005 | .001 | .097 | .050 | .024 | .013 |
| AE | .105 | .050 | .024 | .012 | .203 | .128 | .068 | .043 | .040 | .013 | .004 | .001 | .108 | .058 | .024 | .012 |
| EA | .092 | .048 | .021 | .013 | .187 | .108 | .061 | .038 | .043 | .016 | .004 | .002 | .094 | .045 | .018 | .008 |
| BC | .113 | .059 | .025 | .013 | .179 | .114 | .066 | .032 | .050 | .018 | .004 | .001 | .110 | .050 | .024 | .011 |
| CB | .109 | .054 | .026 | .012 | .205 | .136 | .068 | .045 | .041 | .016 | .004 | .001 | .099 | .048 | .019 | .010 |
| BE | .101 | .050 | .018 | .008 | .205 | .126 | .065 | .043 | .041 | .016 | .006 | .003 | .105 | .052 | .024 | .010 |
| EB | .115 | .054 | .025 | .013 | .190 | .116 | .063 | .042 | .041 | .014 | .004 | .002 | .103 | .056 | .025 | .014 |
| CD | .107 | .055 | .022 | .013 | .192 | .112 | .063 | .041 | .050 | .021 | .006 | .003 | .104 | .055 | .019 | .008 |
| CE | .099 | .053 | .024 | .014 | .201 | .132 | .066 | .042 | .040 | .016 | .005 | .003 | .109 | .058 | .024 | .012 |
| EC | .109 | .060 | .026 | .013 | .205 | .126 | .068 | .039 | .045 | .017 | .004 | .002 | .119 | .058 | .023 | .013 |

Note: A,B,C,D,E refer to forms of heteroscedasticity.

Table 6

Empirical Probability of Inclusion of Each Fixed X in a Region of Significance at Nominal Alpha Levels of .10, .05, .025, and .01 for Each Experimental Condition

| | | ALPHA = | .10 | | | | | .05 | | | | | .025 | | | | | .01 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXED X = | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 |
| | G SIZE | VAR | | | | | | | | | | | | | | | | | | | | |
| AA | 10,10 | 1,1 | .096 | .096 | .100 | .110 | .109 | .048 | .048 | .056 | .054 | .059 | .019 | .020 | .027 | .032 | .033 | .007 | .008 | .010 | .013 | .012 |
| | | 2,1 | .107 | .100 | .101 | .104 | .110 | .053 | .052 | .047 | .052 | .054 | .030 | .027 | .025 | .024 | .023 | .010 | .009 | .010 | .010 | .010 |
| | | 5,1 | .102 | .103 | .109 | .102 | .100 | .058 | .058 | .054 | .053 | .052 | .032 | .031 | .027 | .029 | .027 | .016 | .013 | .012 | .013 | .013 |
| | 10,20 | 1,1 | .100 | .109 | .104 | .101 | .096 | .050 | .055 | .055 | .055 | .046 | .023 | .027 | .029 | .024 | .027 | .010 | .013 | .013 | .010 | .009 |
| | | 2,1 | .155 | .150 | .149 | .144 | .146 | .084 | .082 | .081 | .074 | .077 | .048 | .048 | .042 | .040 | .042 | .020 | .020 | .018 | .020 | .020 |
| | | 5,1 | .202 | .202 | .201 | .201 | .205 | .133 | .127 | .125 | .128 | .129 | .084 | .077 | .079 | .082 | .082 | .042 | .042 | .043 | .044 | .044 |
| | | 1,2 | .072 | .069 | .069 | .078 | .070 | .035 | .032 | .037 | .030 | .029 | .015 | .017 | .017 | .013 | .010 | .005 | .007 | .006 | .003 | .003 |
| | | 1,5 | .041 | .043 | .042 | .044 | .040 | .015 | .015 | .016 | .018 | .018 | .007 | .005 | .006 | .008 | .008 | .001 | .001 | .002 | .002 | .002 |
| | 20,20 | 1,1 | .100 | .096 | .098 | .101 | .100 | .052 | .054 | .045 | .047 | .054 | .028 | .027 | .021 | .023 | .025 | .009 | .011 | .009 | .009 | .009 |
| | | 2,1 | .106 | .104 | .109 | .111 | .109 | .052 | .054 | .055 | .054 | .056 | .025 | .027 | .027 | .029 | .027 | .012 | .012 | .011 | .013 | .013 |
| | | 5,1 | .096 | .094 | .097 | .094 | .093 | .049 | .047 | .050 | .049 | .045 | .025 | .025 | .025 | .025 | .027 | .011 | .008 | .008 | .012 | .012 |
| BB | 10,10 | 1,1 | .045 | .066 | .091 | .071 | .047 | .018 | .031 | .039 | .027 | .018 | .008 | .013 | .021 | .014 | .009 | .003 | .006 | .008 | .007 | .002 |
| | | 2,1 | .049 | .074 | .093 | .069 | .048 | .021 | .034 | .048 | .033 | .019 | .006 | .017 | .022 | .015 | .008 | .002 | .005 | .010 | .004 | .003 |
| | | 5,1 | .049 | .071 | .095 | .074 | .056 | .022 | .031 | .048 | .038 | .024 | .009 | .015 | .024 | .019 | .013 | .004 | .005 | .010 | .008 | .004 |
| | 10,20 | 1,1 | .045 | .069 | .097 | .071 | .050 | .019 | .032 | .045 | .029 | .020 | .007 | .012 | .020 | .013 | .009 | .002 | .005 | .007 | .004 | .004 |
| | | 2,1 | .077 | .106 | .142 | .112 | .082 | .037 | .059 | .080 | .058 | .039 | .014 | .027 | .043 | .029 | .020 | .006 | .010 | .016 | .010 | .005 |
| | | 5,1 | .117 | .151 | .189 | .155 | .116 | .066 | .086 | .110 | .087 | .065 | .034 | .053 | .064 | .051 | .037 | .016 | .023 | .032 | .026 | .016 |
| | | 1,2 | .026 | .040 | .068 | .045 | .027 | .010 | .017 | .024 | .015 | .009 | .004 | .007 | .010 | .006 | .004 | .002 | .003 | .003 | .001 | .000 |
| | | 1,5 | .014 | .025 | .038 | .026 | .013 | .003 | .007 | .012 | .007 | .004 | .001 | .001 | .004 | .002 | .001 | .000 | .000 | .001 | .001 | .000 |
| | 20,20 | 1,1 | .045 | .071 | .095 | .069 | .044 | .019 | .029 | .047 | .032 | .016 | .007 | .014 | .022 | .013 | .007 | .001 | .004 | .008 | .005 | .003 |
| | | 2,1 | .052 | .073 | .097 | .076 | .056 | .021 | .034 | .047 | .038 | .023 | .007 | .015 | .027 | .018 | .009 | .002 | .005 | .010 | .007 | .002 |
| | | 5,1 | .054 | .074 | .093 | .069 | .054 | .022 | .034 | .046 | .033 | .022 | .010 | .014 | .022 | .015 | .010 | .002 | .005 | .008 | .006 | .004 |
| CC | 10,10 | 1,1 | .038 | .038 | .103 | .170 | .179 | .016 | .011 | .050 | .102 | .104 | .006 | .005 | .026 | .061 | .063 | .001 | .001 | .008 | .026 | .030 |
| | | 2,1 | .029 | .030 | .100 | .173 | .172 | .009 | .013 | .051 | .099 | .105 | .003 | .005 | .027 | .057 | .057 | .001 | .001 | .010 | .028 | .028 |
| | | 5,1 | .040 | .040 | .102 | .174 | .179 | .017 | .018 | .057 | .106 | .111 | .000 | .000 | .000 | .000 | .000 | .006 | .007 | .032 | .069 | .066 |
| | 10,20 | 1,1 | .003 | .003 | .014 | .032 | .036 | .011 | .010 | .049 | .103 | .103 | .003 | .003 | .023 | .058 | .062 | .000 | .000 | .009 | .029 | .030 |
| | | 2,1 | .055 | .054 | .146 | .227 | .229 | .026 | .025 | .086 | .150 | .148 | .011 | .010 | .055 | .097 | .097 | .005 | .004 | .023 | .056 | .062 |
| | | 5,1 | .104 | .098 | .193 | .270 | .272 | .050 | .049 | .124 | .192 | .192 | .023 | .024 | .077 | .136 | .138 | .006 | .011 | .044 | .086 | .089 |
| | | 1,2 | .019 | .014 | .066 | .116 | .123 | .004 | .004 | .026 | .063 | .066 | .001 | .000 | .011 | .032 | .035 | .000 | .000 | .002 | .013 | .015 |
| | | 1,5 | .010 | .007 | .040 | .088 | .039 | .001 | .002 | .013 | .042 | .041 | .001 | .001 | .005 | .018 | .019 | .000 | .000 | .001 | .008 | .007 |
| | 20,20 | 1,1 | .034 | .032 | .099 | .164 | .163 | .012 | .010 | .050 | .097 | .101 | .004 | .004 | .024 | .054 | .055 | .001 | .001 | .009 | .028 | .030 |
| | | 2,1 | .035 | .035 | .098 | .172 | .177 | .013 | .012 | .052 | .098 | .104 | .005 | .006 | .026 | .061 | .064 | .002 | .003 | .009 | .032 | .032 |
| | | 5,1 | .031 | .031 | .106 | .191 | .191 | .010 | .008 | .058 | .112 | .110 | .004 | .001 | .029 | .067 | .064 | .000 | .001 | .011 | .034 | .034 |

## Table 6--Continued

| | | ALPHA = | .10 | | | | | .05 | | | | | .025 | | | | | .01 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXED X = | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 |
| | G SIZE | VAR | | | | | | | | | | | | | | | | | | | | |
| DD | 10,10 | 1,1 | .174 | .177 | .105 | .038 | .036 | .106 | .105 | .052 | .015 | .017 | .060 | .057 | .025 | .007 | .006 | .026 | .028 | .009 | .002 | .001 |
| | | 2,1 | .170 | .171 | .108 | .036 | .037 | .100 | .106 | .059 | .016 | .015 | .059 | .059 | .031 | .007 | .006 | .027 | .030 | .013 | .003 | .002 |
| | | 5,1 | .170 | .168 | .107 | .043 | .043 | .104 | .106 | .061 | .016 | .016 | .056 | .068 | .035 | .008 | .005 | .036 | .035 | .018 | .004 | .002 |
| | 10,20 | 1,1 | .169 | .164 | .100 | .033 | .033 | .099 | .100 | .052 | .010 | .013 | .058 | .058 | .026 | .004 | .004 | .031 | .031 | .011 | .001 | .000 |
| | | 2,1 | .232 | .235 | .149 | .057 | .062 | .154 | .152 | .081 | .024 | .025 | .103 | .099 | .048 | .011 | .010 | .054 | .052 | .021 | .003 | .003 |
| | | 5,1 | .232 | .283 | .206 | .094 | .100 | .210 | .199 | .137 | .048 | .049 | .146 | .147 | .090 | .024 | .023 | .091 | .094 | .051 | .012 | .010 |
| | | 1,2 | .121 | .129 | .074 | .022 | .019 | .069 | .068 | .033 | .008 | .006 | .041 | .042 | .021 | .003 | .001 | .017 | .019 | .006 | .001 | .001 |
| | | 1,5 | .091 | .089 | .045 | .009 | .007 | .045 | .045 | .017 | .002 | .002 | .022 | .021 | .007 | .001 | .000 | .008 | .007 | .001 | .000 | .000 |
| | 20,20 | 1,1 | .173 | .180 | .107 | .033 | .033 | .109 | .107 | .047 | .011 | .010 | .071 | .066 | .027 | .004 | .003 | .032 | .035 | .014 | .002 | .000 |
| | | 2,1 | .159 | .177 | .104 | .039 | .039 | .108 | .104 | .053 | .018 | .015 | .064 | .064 | .029 | .006 | .005 | .033 | .031 | .013 | .002 | .001 |
| | | 5,1 | .179 | .185 | .108 | .034 | .035 | .105 | .105 | .055 | .012 | .011 | .065 | .064 | .029 | .005 | .003 | .035 | .036 | .011 | .002 | .001 |
| EE | 10,10 | 1,1 | .155 | .133 | .093 | .117 | .136 | .090 | .071 | .050 | .061 | .078 | .051 | .040 | .026 | .033 | .045 | .027 | .016 | .011 | .017 | .022 |
| | | 2,1 | .149 | .134 | .115 | .138 | .156 | .088 | .081 | .060 | .071 | .086 | .049 | .038 | .026 | .034 | .041 | .022 | .016 | .009 | .013 | .017 |
| | | 5,1 | .151 | .132 | .107 | .130 | .152 | .090 | .071 | .055 | .078 | .087 | .052 | .039 | .028 | .042 | .050 | .025 | .019 | .011 | .019 | .023 |
| | 10,20 | 1,1 | .138 | .117 | .094 | .118 | .133 | .075 | .058 | .049 | .057 | .077 | .041 | .033 | .024 | .031 | .041 | .020 | .015 | .009 | .013 | .018 |
| | | 2,1 | .190 | .176 | .157 | .179 | .208 | .125 | .104 | .085 | .112 | .128 | .034 | .067 | .049 | .071 | .083 | .046 | .036 | .024 | .033 | .043 |
| | | 5,1 | .258 | .235 | .217 | .247 | .265 | .176 | .156 | .139 | .154 | .178 | .119 | .100 | .086 | .100 | .121 | .069 | .057 | .046 | .063 | .076 |
| | | 1,2 | .091 | .084 | .066 | .079 | .091 | .046 | .040 | .029 | .035 | .047 | .024 | .017 | .011 | .015 | .022 | .011 | .009 | .004 | .005 | .006 |
| | | 1,5 | .067 | .055 | .041 | .062 | .069 | .032 | .022 | .010 | .024 | .034 | .013 | .009 | .005 | .007 | .012 | .004 | .002 | .001 | .002 | .005 |
| | 20,20 | 1,1 | .143 | .120 | .102 | .119 | .147 | .083 | .066 | .048 | .064 | .079 | .043 | .035 | .027 | .037 | .042 | .020 | .013 | .011 | .016 | .023 |
| | | 2,1 | .145 | .121 | .104 | .125 | .135 | .076 | .060 | .052 | .067 | .082 | .042 | .033 | .025 | .034 | .050 | .021 | .016 | .011 | .015 | .026 |
| | | 5,1 | .150 | .133 | .106 | .123 | .138 | .091 | .074 | .054 | .067 | .076 | .051 | .043 | .026 | .031 | .041 | .024 | .020 | .013 | .015 | .019 |
| AB | 10,10 | 1,1 | .058 | .074 | .087 | .082 | .071 | .032 | .038 | .038 | .035 | .034 | .014 | .018 | .019 | .018 | .014 | .004 | .004 | .007 | .007 | .005 |
| | | 2,1 | .077 | .089 | .102 | .091 | .085 | .036 | .047 | .050 | .050 | .042 | .020 | .021 | .027 | .022 | .018 | .009 | .011 | .010 | .009 | .007 |
| | | 5,1 | .103 | .106 | .107 | .111 | .105 | .055 | .054 | .056 | .059 | .056 | .028 | .030 | .035 | .033 | .029 | .011 | .013 | .016 | .015 | .014 |
| | 10,20 | 1,1 | .093 | .104 | .107 | .094 | .087 | .048 | .050 | .057 | .048 | .037 | .025 | .027 | .031 | .023 | .022 | .009 | .014 | .013 | .010 | .010 |
| | | 2,1 | .121 | .123 | .132 | .142 | .135 | .064 | .068 | .077 | .072 | .072 | .037 | .037 | .042 | .041 | .038 | .017 | .015 | .017 | .016 | .017 |
| | | 5,1 | .189 | .206 | .208 | .208 | .203 | .117 | .130 | .130 | .128 | .124 | .072 | .077 | .083 | .082 | .077 | .039 | .046 | .051 | .046 | .043 |
| | | 1,2 | .042 | .050 | .059 | .061 | .054 | .019 | .022 | .025 | .023 | .020 | .008 | .010 | .012 | .011 | .008 | .003 | .004 | .005 | .003 | .002 |
| | | 1,5 | .021 | .027 | .040 | .033 | .023 | .005 | .006 | .012 | .012 | .008 | .001 | .001 | .003 | .004 | .003 | .000 | .000 | .000 | .000 | .001 |
| | 20,20 | 1,1 | .091 | .093 | .108 | .098 | .083 | .034 | .042 | .056 | .047 | .039 | .014 | .017 | .029 | .023 | .017 | .006 | .008 | .012 | .007 | .008 |
| | | 2,1 | .079 | .088 | .096 | .097 | .091 | .036 | .040 | .048 | .045 | .041 | .020 | .020 | .027 | .025 | .020 | .007 | .008 | .010 | .010 | .008 |
| | | 5,1 | .100 | .109 | .099 | .092 | .093 | .051 | .054 | .051 | .042 | .040 | .029 | .023 | .028 | .022 | .019 | .012 | .009 | .010 | .008 | .008 |

Table 6--Continued

| | | ALPHA = | .10 | | | | | .05 | | | | | .025 | | | | | .01 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXED X = | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 |
| | G SIZE | VAR | | | | | | | | | | | | | | | | | | | | |
| | 10,10 | 2,1 | .058 | .083 | .096 | .080 | .064 | .028 | .036 | .045 | .032 | .030 | .014 | .018 | .021 | .013 | .011 | .005 | .007 | .007 | .005 | .005 |
| | | 5,1 | .059 | .072 | .088 | .077 | .060 | .027 | .033 | .041 | .032 | .025 | .015 | .017 | .019 | .011 | .010 | .005 | .005 | .008 | .005 | .004 |
| | 10,20 | 1,1 | .064 | .063 | .087 | .075 | .060 | .031 | .035 | .041 | .032 | .030 | .011 | .016 | .020 | .019 | .013 | .004 | .006 | .007 | .007 | .004 |
| BA | | 2,1 | .089 | .109 | .133 | .114 | .092 | .045 | .056 | .075 | .058 | .043 | .021 | .030 | .040 | .032 | .022 | .008 | .013 | .020 | .015 | .006 |
| | | 5,1 | .136 | .162 | .191 | .166 | .133 | .073 | .098 | .118 | .099 | .071 | .043 | .058 | .073 | .057 | .035 | .018 | .028 | .039 | .027 | .017 |
| | | 1,2 | .044 | .054 | .061 | .050 | .042 | .017 | .024 | .026 | .019 | .017 | .008 | .009 | .012 | .007 | .006 | .003 | .003 | .004 | .003 | .002 |
| | | 1,5 | .033 | .034 | .045 | .035 | .031 | .010 | .015 | .019 | .010 | .012 | .005 | .005 | .007 | .005 | .002 | .001 | .002 | .003 | .001 | .001 |
| | 20,20 | 2,1 | .063 | .078 | .089 | .082 | .071 | .030 | .036 | .045 | .036 | .031 | .013 | .015 | .019 | .014 | .012 | .004 | .005 | .008 | .006 | .004 |
| | | 5,1 | .055 | .075 | .097 | .081 | .058 | .025 | .037 | .048 | .037 | .025 | .012 | .019 | .026 | .016 | .012 | .004 | .007 | .010 | .006 | .004 |
| | 10,10 | 1,1 | .066 | .068 | .105 | .140 | .138 | .031 | .030 | .053 | .078 | .076 | .011 | .011 | .028 | .041 | .044 | .004 | .005 | .013 | .023 | .022 |
| | | 2,1 | .070 | .078 | .100 | .116 | .120 | .033 | .034 | .051 | .067 | .063 | .016 | .013 | .024 | .036 | .035 | .005 | .005 | .010 | .015 | .016 |
| | | 5,1 | .100 | .102 | .111 | .117 | .115 | .053 | .050 | .056 | .065 | .065 | .031 | .029 | .034 | .035 | .036 | .016 | .014 | .016 | .017 | .016 |
| | 10,20 | 1,1 | .074 | .073 | .106 | .128 | .125 | .033 | .033 | .048 | .072 | .070 | .017 | .017 | .023 | .040 | .038 | .006 | .005 | .010 | .018 | .017 |
| AC | | 2,1 | .130 | .124 | .143 | .167 | .169 | .075 | .072 | .082 | .098 | .100 | .042 | .040 | .049 | .056 | .058 | .020 | .019 | .022 | .029 | .024 |
| | | 5,1 | .196 | .197 | .205 | .205 | .219 | .121 | .124 | .129 | .140 | .138 | .079 | .082 | .087 | .086 | .087 | .044 | .045 | .046 | .051 | .048 |
| | | 1,2 | .042 | .042 | .067 | .103 | .095 | .016 | .015 | .029 | .048 | .051 | .008 | .006 | .014 | .024 | .027 | .002 | .002 | .003 | .012 | .012 |
| | | 1,5 | .016 | .021 | .051 | .082 | .080 | .004 | .007 | .018 | .039 | .038 | .001 | .002 | .006 | .015 | .017 | .000 | .000 | .002 | .008 | .007 |
| | 20,20 | 1,1 | .059 | .066 | .113 | .146 | .143 | .026 | .030 | .057 | .084 | .079 | .011 | .012 | .030 | .047 | .043 | .005 | .006 | .012 | .023 | .022 |
| | | 2,1 | .073 | .076 | .105 | .139 | .139 | .037 | .036 | .058 | .078 | .076 | .019 | .018 | .028 | .041 | .038 | .008 | .007 | .012 | .020 | .017 |
| | | 5,1 | .099 | .097 | .107 | .119 | .114 | .048 | .046 | .054 | .063 | .063 | .022 | .023 | .025 | .033 | .037 | .008 | .008 | .011 | .016 | .014 |
| | 10,10 | 2,1 | .054 | .054 | .106 | .153 | .159 | .026 | .024 | .052 | .087 | .089 | .009 | .009 | .029 | .047 | .049 | .003 | .003 | .013 | .026 | .021 |
| | | 5,1 | .054 | .050 | .104 | .161 | .162 | .025 | .024 | .057 | .093 | .096 | .010 | .013 | .031 | .056 | .057 | .005 | .004 | .012 | .025 | .028 |
| | 10,20 | 1,1 | .054 | .052 | .090 | .142 | .144 | .017 | .019 | .048 | .076 | .077 | .009 | .008 | .024 | .042 | .043 | .003 | .002 | .011 | .019 | .021 |
| CA | | 2,1 | .078 | .075 | .141 | .202 | .206 | .034 | .035 | .085 | .126 | .125 | .014 | .017 | .046 | .081 | .082 | .006 | .006 | .020 | .047 | .047 |
| | | 5,1 | .105 | .116 | .205 | .279 | .286 | .058 | .058 | .132 | .192 | .194 | .028 | .031 | .091 | .137 | .138 | .013 | .013 | .053 | .094 | .092 |
| | | 1,2 | .041 | .040 | .071 | .086 | .086 | .016 | .016 | .031 | .042 | .039 | .005 | .008 | .015 | .021 | .020 | .001 | .003 | .005 | .008 | .007 |
| | | 1,5 | .028 | .026 | .038 | .063 | .065 | .011 | .008 | .015 | .028 | .026 | .004 | .004 | .007 | .010 | .009 | .001 | .000 | .001 | .002 | .002 |
| | 20,20 | 2,1 | .057 | .051 | .097 | .149 | .151 | .020 | .021 | .049 | .084 | .087 | .008 | .008 | .023 | .047 | .051 | .001 | .002 | .006 | .023 | .021 |
| | | 5,1 | .043 | .048 | .096 | .151 | .151 | .015 | .020 | .049 | .088 | .095 | .006 | .006 | .028 | .052 | .048 | .002 | .002 | .013 | .025 | .021 |

Table 6--Continued

| | G SIZE | VAR | .10 X1 | .10 X2 | .10 X3 | .10 X4 | .10 X5 | .05 X1 | .05 X2 | .05 X3 | .05 X4 | .05 X5 | .025 X1 | .025 X2 | .025 X3 | .025 X4 | .025 X5 | .01 X1 | .01 X2 | .01 X3 | .01 X4 | .01 X5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AE | 10,10 | 1,1 | .130 | .118 | .108 | .112 | .122 | .068 | .065 | .051 | .060 | .067 | .039 | .037 | .027 | .028 | .033 | .018 | .015 | .010 | .010 | .013 |
| | | 2,1 | .117 | .114 | .104 | .109 | .111 | .064 | .065 | .050 | .054 | .062 | .035 | .027 | .027 | .033 | .040 | .016 | .012 | .011 | .014 | .018 |
| | | 5,1 | .110 | .112 | .105 | .105 | .110 | .059 | .058 | .051 | .053 | .052 | .031 | .029 | .029 | .027 | .029 | .013 | .012 | .013 | .013 | .013 |
| | 10,20 | 1,1 | .120 | .113 | .093 | .103 | .111 | .062 | .056 | .047 | .051 | .059 | .034 | .029 | .022 | .024 | .032 | .014 | .010 | .010 | .008 | .014 |
| | | 2,1 | .154 | .154 | .142 | .142 | .148 | .095 | .088 | .081 | .077 | .087 | .054 | .047 | .044 | .042 | .048 | .028 | .026 | .020 | .021 | .023 |
| | | 5,1 | .207 | .205 | .208 | .220 | .224 | .129 | .128 | .135 | .137 | .139 | .084 | .078 | .084 | .087 | .095 | .047 | .045 | .046 | .047 | .054 |
| | | 1,2 | .085 | .074 | .071 | .082 | .092 | .035 | .031 | .031 | .036 | .039 | .015 | .015 | .015 | .016 | .018 | .007 | .007 | .005 | .007 | .007 |
| | | 1,5 | .064 | .052 | .039 | .052 | .062 | .025 | .018 | .013 | .020 | .027 | .009 | .008 | .005 | .008 | .012 | .003 | .002 | .001 | .002 | .004 |
| | 20,20 | 1,1 | .116 | .115 | .102 | .111 | .123 | .067 | .059 | .050 | .057 | .063 | .034 | .028 | .022 | .026 | .033 | .015 | .011 | .010 | .011 | .012 |
| | | 2,1 | .116 | .109 | .102 | .118 | .116 | .057 | .052 | .046 | .061 | .064 | .033 | .025 | .021 | .027 | .030 | .012 | .010 | .009 | .009 | .012 |
| | | 5,1 | .112 | .116 | .109 | .108 | .111 | .059 | .061 | .059 | .055 | .058 | .034 | .029 | .031 | .029 | .033 | .015 | .015 | .013 | .015 | .016 |
| EA | 10,10 | 2,1 | .141 | .124 | .098 | .117 | .130 | .080 | .067 | .049 | .061 | .080 | .044 | .034 | .026 | .033 | .042 | .022 | .017 | .010 | .014 | .019 |
| | | 5,1 | .142 | .125 | .095 | .123 | .141 | .080 | .068 | .050 | .064 | .079 | .051 | .036 | .027 | .038 | .042 | .024 | .015 | .013 | .018 | .022 |
| | 10,20 | 1,1 | .131 | .120 | .103 | .112 | .132 | .072 | .062 | .051 | .056 | .070 | .041 | .033 | .024 | .027 | .032 | .018 | .014 | .010 | .011 | .014 |
| | | 2,1 | .181 | .164 | .144 | .151 | .170 | .112 | .103 | .082 | .091 | .104 | .074 | .063 | .046 | .054 | .061 | .038 | .032 | .020 | .023 | .031 |
| | | 5,1 | .246 | .219 | .194 | .225 | .253 | .165 | .147 | .120 | .141 | .164 | .109 | .097 | .077 | .089 | .109 | .069 | .055 | .040 | .052 | .067 |
| | | 1,2 | .079 | .076 | .071 | .075 | .085 | .044 | .035 | .031 | .036 | .039 | .022 | .017 | .013 | .018 | .018 | .008 | .005 | .004 | .005 | .007 |
| | | 1,5 | .052 | .046 | .040 | .046 | .052 | .017 | .015 | .015 | .017 | .021 | .007 | .006 | .005 | .007 | .007 | .002 | .001 | .001 | .001 | .002 |
| | 20,20 | 2,1 | .123 | .113 | .108 | .115 | .119 | .063 | .058 | .055 | .060 | .069 | .033 | .029 | .027 | .031 | .040 | .015 | .015 | .009 | .014 | .016 |
| | | 5,1 | .139 | .120 | .096 | .119 | .135 | .075 | .062 | .046 | .060 | .076 | .045 | .035 | .024 | .031 | .040 | .022 | .016 | .008 | .013 | .016 |
| BC | 10,10 | 1,1 | .039 | .053 | .097 | .115 | .107 | .016 | .021 | .051 | .063 | .057 | .006 | .009 | .022 | .032 | .031 | .002 | .004 | .009 | .014 | .011 |
| | | 2,1 | .047 | .065 | .101 | .107 | .093 | .022 | .027 | .051 | .056 | .048 | .008 | .012 | .026 | .029 | .024 | .003 | .005 | .011 | .015 | .010 |
| | | 5,1 | .058 | .076 | .110 | .099 | .078 | .022 | .031 | .057 | .051 | .040 | .008 | .015 | .027 | .029 | .018 | .002 | .005 | .013 | .011 | .008 |
| | 10,20 | 1,1 | .041 | .049 | .088 | .103 | .091 | .016 | .020 | .041 | .049 | .044 | .006 | .009 | .019 | .022 | .018 | .001 | .002 | .006 | .007 | .005 |
| | | 2,1 | .075 | .097 | .141 | .133 | .107 | .034 | .050 | .077 | .075 | .061 | .015 | .026 | .042 | .043 | .034 | .006 | .009 | .019 | .023 | .017 |
| | | 5,1 | .121 | .148 | .180 | .174 | .142 | .063 | .079 | .111 | .098 | .079 | .029 | .044 | .065 | .057 | .043 | .014 | .021 | .032 | .027 | .020 |
| | | 1,2 | .023 | .032 | .064 | .087 | .075 | .005 | .008 | .027 | .042 | .035 | .002 | .004 | .013 | .019 | .017 | .000 | .001 | .005 | .007 | .006 |
| | | 1,5 | .010 | .014 | .053 | .071 | .062 | .001 | .003 | .018 | .031 | .028 | .000 | .001 | .005 | .016 | .013 | .000 | .000 | .001 | .005 | .004 |
| | 20,20 | 1,1 | .040 | .045 | .088 | .119 | .110 | .014 | .018 | .045 | .058 | .055 | .006 | .007 | .021 | .032 | .028 | .001 | .003 | .007 | .011 | .010 |
| | | 2,1 | .051 | .071 | .105 | .113 | .101 | .021 | .028 | .054 | .064 | .054 | .007 | .011 | .028 | .035 | .026 | .001 | .003 | .011 | .012 | .011 |
| | | 5,1 | .047 | .065 | .108 | .094 | .076 | .019 | .029 | .050 | .051 | .038 | .009 | .012 | .027 | .027 | .018 | .002 | .005 | .011 | .012 | .008 |

Table 6--Continued

| | | ALPHA = | .10 | | | | | .05 | | | | | .025 | | | | | .01 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXED X = | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 |
| | G SIZE | VAR | | | | | | | | | | | | | | | | | | | | |
| CB | 10,10 | 2,1 | .039 | .044 | .099 | .127 | .117 | .014 | .018 | .050 | .068 | .062 | .005 | .006 | .023 | .039 | .034 | .001 | .001 | .006 | .013 | .013 |
| | | 5,1 | .040 | .043 | .110 | .166 | .165 | .016 | .019 | .058 | .102 | .097 | .008 | .008 | .035 | .058 | .056 | .002 | .004 | .014 | .030 | .029 |
| | 10,20 | 1,1 | .041 | .050 | .097 | .131 | .124 | .016 | .016 | .051 | .068 | .062 | .007 | .006 | .023 | .036 | .035 | .002 | .002 | .008 | .015 | .014 |
| | | 2,1 | .062 | .070 | .152 | .201 | .197 | .027 | .029 | .086 | .130 | .124 | .009 | .014 | .050 | .083 | .079 | .005 | .005 | .021 | .046 | .044 |
| | | 5,1 | .103 | .104 | .212 | .277 | .275 | .053 | .052 | .141 | .206 | .195 | .030 | .028 | .088 | .144 | .143 | .014 | .013 | .053 | .094 | .091 |
| | | 1,2 | .021 | .025 | .068 | .090 | .080 | .006 | .009 | .033 | .043 | .036 | .001 | .002 | .016 | .022 | .018 | .000 | .000 | .005 | .010 | .007 |
| | | 1,5 | .013 | .016 | .040 | .045 | .036 | .004 | .005 | .015 | .017 | .012 | .001 | .003 | .005 | .008 | .004 | .000 | .001 | .001 | .002 | .001 |
| | 20,20 | 2,1 | .042 | .045 | .099 | .145 | .132 | .013 | .019 | .051 | .077 | .073 | .006 | .009 | .026 | .040 | .038 | .002 | .003 | .010 | .017 | .017 |
| | | 5,1 | .036 | .040 | .100 | .154 | .160 | .013 | .017 | .049 | .084 | .084 | .005 | .006 | .025 | .049 | .047 | .001 | .002 | .010 | .025 | .026 |
| BE | 10,10 | 1,1 | .093 | .039 | .090 | .090 | .093 | .040 | .039 | .038 | .049 | .051 | .019 | .019 | .016 | .022 | .024 | .007 | .006 | .007 | .010 | .009 |
| | | 2,1 | .039 | .094 | .095 | .094 | .088 | .049 | .049 | .052 | .047 | .044 | .024 | .028 | .029 | .023 | .019 | .010 | .010 | .014 | .009 | .009 |
| | | 5,1 | .070 | .085 | .100 | .088 | .073 | .036 | .042 | .048 | .038 | .030 | .016 | .022 | .021 | .017 | .013 | .006 | .008 | .007 | .007 | .005 |
| | 10,20 | 1,1 | .036 | .098 | .095 | .092 | .082 | .046 | .046 | .050 | .049 | .038 | .019 | .024 | .023 | .019 | .017 | .009 | .009 | .009 | .007 | .004 |
| | | 2,1 | .105 | .126 | .142 | .116 | .098 | .051 | .064 | .076 | .062 | .052 | .026 | .031 | .036 | .032 | .026 | .011 | .011 | .016 | .012 | .011 |
| | | 5,1 | .131 | .168 | .205 | .137 | .153 | .073 | .097 | .128 | .115 | .088 | .044 | .061 | .078 | .070 | .052 | .021 | .029 | .042 | .038 | .027 |
| | | 1,2 | .051 | .068 | .064 | .066 | .061 | .027 | .028 | .029 | .028 | .025 | .008 | .010 | .014 | .013 | .011 | .003 | .003 | .004 | .005 | .004 |
| | | 1,5 | .052 | .056 | .041 | .046 | .054 | .024 | .021 | .017 | .018 | .023 | .011 | .009 | .006 | .009 | .010 | .003 | .003 | .002 | .003 | .004 |
| | 20,20 | 1,1 | .102 | .093 | .095 | .088 | .035 | .050 | .050 | .044 | .046 | .046 | .024 | .025 | .023 | .026 | .024 | .011 | .010 | .010 | .011 | .010 |
| | | 2,1 | .058 | .079 | .097 | .090 | .079 | .026 | .036 | .046 | .044 | .036 | .013 | .017 | .025 | .022 | .016 | .006 | .006 | .009 | .007 | .006 |
| | | 5,1 | .071 | .083 | .105 | .097 | .080 | .033 | .043 | .053 | .046 | .041 | .014 | .020 | .029 | .022 | .016 | .003 | .008 | .010 | .010 | .006 |
| EB | 10,10 | 2,1 | .115 | .115 | .107 | .105 | .108 | .056 | .057 | .056 | .053 | .058 | .030 | .026 | .026 | .028 | .033 | .012 | .009 | .008 | .012 | .013 |
| | | 5,1 | .143 | .129 | .118 | .126 | .141 | .081 | .071 | .059 | .066 | .075 | .043 | .037 | .031 | .038 | .043 | .021 | .019 | .016 | .018 | .022 |
| | 10,20 | 1,1 | .121 | .109 | .092 | .104 | .107 | .061 | .059 | .042 | .053 | .059 | .030 | .028 | .021 | .028 | .034 | .012 | .010 | .006 | .010 | .012 |
| | | 2,1 | .191 | .175 | .153 | .164 | .174 | .116 | .103 | .085 | .099 | .106 | .071 | .062 | .048 | .054 | .067 | .034 | .029 | .022 | .029 | .038 |
| | | 5,1 | .258 | .234 | .199 | .214 | .236 | .175 | .152 | .125 | .145 | .159 | .120 | .100 | .077 | .091 | .104 | .070 | .060 | .047 | .055 | .060 |
| | | 1,2 | .070 | .062 | .065 | .060 | .060 | .029 | .025 | .029 | .027 | .026 | .011 | .010 | .009 | .013 | .013 | .004 | .004 | .002 | .005 | .005 |
| | | 1,5 | .031 | .036 | .038 | .035 | .035 | .010 | .014 | .013 | .013 | .012 | .003 | .004 | .005 | .007 | .005 | .001 | .001 | .002 | .001 | .001 |
| | 20,20 | 2,1 | .109 | .103 | .092 | .108 | .121 | .061 | .052 | .046 | .055 | .069 | .033 | .027 | .022 | .032 | .036 | .015 | .012 | .008 | .013 | .016 |
| | | 5,1 | .125 | .113 | .105 | .132 | .140 | .070 | .063 | .059 | .071 | .082 | .041 | .035 | .030 | .040 | .049 | .020 | .017 | .014 | .018 | .025 |

Table 6--Continued

| | | | ALPHA = | | .10 | | | | | .05 | | | | | .025 | | | | | .01 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXED X = | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 | X1 | X2 | X3 | X4 | X5 |
| | G SIZE | VAR | | | | | | | | | | | | | | | | | | | | |
| CD | 10,10 | 1,1 | .103 | .099 | .104 | .105 | .114 | .054 | .052 | .053 | .057 | .057 | .025 | .028 | .027 | .030 | .031 | .012 | .014 | .011 | .013 | .014 |
| | | 2,1 | .079 | .088 | .107 | .126 | .120 | .041 | .039 | .057 | .067 | .063 | .020 | .019 | .025 | .036 | .033 | .007 | .007 | .009 | .014 | .015 |
| | | 5,1 | .056 | .059 | .108 | .153 | .158 | .025 | .025 | .060 | .092 | .096 | .011 | .011 | .031 | .057 | .060 | .004 | .005 | .015 | .028 | .027 |
| | 10,20 | 1,1 | .050 | .079 | .096 | .126 | .125 | .037 | .038 | .050 | .067 | .070 | .020 | .020 | .027 | .038 | .037 | .009 | .009 | .011 | .013 | .014 |
| | | 2,1 | .090 | .095 | .138 | .189 | .193 | .042 | .043 | .079 | .119 | .118 | .020 | .021 | .042 | .072 | .070 | .009 | .007 | .020 | .034 | .037 |
| | | 5,1 | .116 | .117 | .202 | .267 | .270 | .064 | .059 | .119 | .182 | .186 | .033 | .032 | .074 | .130 | .127 | .014 | .014 | .045 | .079 | .079 |
| | | 1,2 | .070 | .071 | .070 | .067 | .074 | .032 | .030 | .028 | .023 | .029 | .016 | .017 | .011 | .013 | .015 | .005 | .006 | .004 | .004 | .004 |
| | | 1,5 | .060 | .065 | .047 | .027 | .028 | .027 | .028 | .021 | .011 | .012 | .014 | .013 | .008 | .005 | .005 | .005 | .004 | .003 | .002 | .001 |
| | 20,20 | 1,1 | .106 | .108 | .108 | .105 | .102 | .052 | .057 | .057 | .051 | .055 | .025 | .026 | .028 | .028 | .029 | .009 | .010 | .009 | .011 | .011 |
| | | 2,1 | .081 | .085 | .102 | .119 | .124 | .043 | .041 | .051 | .073 | .074 | .021 | .022 | .029 | .042 | .044 | .009 | .009 | .016 | .018 | .017 |
| | | 5,1 | .051 | .052 | .106 | .156 | .162 | .020 | .021 | .054 | .094 | .094 | .009 | .010 | .027 | .057 | .059 | .003 | .003 | .012 | .027 | .027 |
| CE | 10,10 | 1,1 | .091 | .081 | .107 | .153 | .159 | .040 | .035 | .053 | .084 | .093 | .020 | .015 | .028 | .045 | .052 | .008 | .004 | .010 | .020 | .024 |
| | | 2,1 | .074 | .063 | .098 | .159 | .161 | .032 | .027 | .049 | .087 | .100 | .017 | .013 | .022 | .048 | .053 | .007 | .005 | .011 | .022 | .024 |
| | | 5,1 | .061 | .056 | .101 | .161 | .175 | .026 | .025 | .053 | .092 | .097 | .010 | .012 | .030 | .055 | .058 | .004 | .004 | .015 | .029 | .032 |
| | 10,20 | 1,1 | .076 | .064 | .104 | .157 | .167 | .034 | .026 | .053 | .090 | .094 | .015 | .011 | .027 | .050 | .056 | .005 | .004 | .013 | .024 | .029 |
| | | 2,1 | .090 | .083 | .145 | .205 | .208 | .044 | .039 | .081 | .128 | .128 | .021 | .017 | .050 | .081 | .083 | .007 | .007 | .023 | .044 | .049 |
| | | 5,1 | .107 | .110 | .207 | .291 | .288 | .055 | .058 | .139 | .206 | .202 | .030 | .030 | .083 | .142 | .142 | .013 | .013 | .048 | .090 | .088 |
| | | 1,2 | .050 | .044 | .067 | .107 | .119 | .025 | .017 | .027 | .053 | .063 | .010 | .005 | .009 | .026 | .028 | .003 | .003 | .003 | .008 | .014 |
| | | 1,5 | .049 | .039 | .043 | .066 | .077 | .020 | .013 | .015 | .026 | .036 | .007 | .005 | .007 | .010 | .014 | .002 | .002 | .002 | .003 | .005 |
| | 20,20 | 1,1 | .093 | .084 | .102 | .140 | .159 | .044 | .035 | .049 | .075 | .086 | .018 | .015 | .023 | .044 | .047 | .007 | .006 | .009 | .020 | .022 |
| | | 2,1 | .057 | .056 | .108 | .164 | .166 | .029 | .022 | .055 | .098 | .100 | .014 | .010 | .028 | .062 | .064 | .005 | .004 | .013 | .034 | .033 |
| | | 5,1 | .058 | .053 | .112 | .168 | .171 | .023 | .020 | .061 | .105 | .102 | .012 | .010 | .029 | .065 | .064 | .002 | .005 | .012 | .033 | .034 |
| EC | 10,10 | 2,1 | .103 | .093 | .106 | .140 | .154 | .055 | .047 | .051 | .081 | .090 | .027 | .023 | .023 | .044 | .053 | .010 | .007 | .010 | .019 | .019 |
| | | 5,1 | .143 | .132 | .112 | .146 | .168 | .082 | .066 | .064 | .081 | .106 | .046 | .034 | .034 | .046 | .062 | .023 | .015 | .013 | .021 | .031 |
| | 10,20 | 1,1 | .105 | .092 | .102 | .132 | .148 | .052 | .038 | .052 | .075 | .083 | .028 | .019 | .026 | .041 | .046 | .012 | .009 | .009 | .021 | .020 |
| | | 2,1 | .135 | .169 | .148 | .173 | .192 | .108 | .093 | .087 | .104 | .116 | .067 | .053 | .047 | .064 | .078 | .035 | .028 | .022 | .030 | .041 |
| | | 5,1 | .257 | .230 | .215 | .243 | .268 | .170 | .153 | .136 | .161 | .180 | .119 | .098 | .086 | .110 | .122 | .070 | .056 | .044 | .062 | .077 |
| | | 1,2 | .059 | .048 | .067 | .108 | .117 | .025 | .021 | .031 | .052 | .059 | .011 | .007 | .014 | .028 | .033 | .003 | .002 | .005 | .012 | .012 |
| | | 1,5 | .018 | .021 | .046 | .079 | .077 | .007 | .007 | .017 | .040 | .039 | .002 | .002 | .006 | .016 | .019 | .001 | .000 | .002 | .006 | .007 |
| | 20,20 | 2,1 | .097 | .079 | .091 | .139 | .153 | .049 | .041 | .047 | .083 | .095 | .027 | .018 | .023 | .046 | .055 | .010 | .005 | .009 | .018 | .026 |
| | | 5,1 | .131 | .120 | .120 | .143 | .157 | .079 | .062 | .061 | .084 | .091 | .040 | .031 | .032 | .047 | .053 | .019 | .015 | .012 | .022 | .028 |

Note: A, B, C, D, E refer to forms of heteroscedasticity.

Table 7

Empirical Probability of Obtaining A Johnson-Neyman Region of Significance
at Nominal Alpha Equal to .10, .05, .025, and .01
for Each Simulated Experimental Condition

| GROUP SIZE 1 2 | VAR 1 2 | ALPHA = .10 | .05 | .025 | .01 | .10 | .05 | .025 | .01 | .10 | .05 | .025 | .01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | AA | | | | BB | | | | CC | | | |
| 10,10 | 1,1 | .246 | .145 | .080 | .035 | .156 | .071 | .033 | .014 | .248 | .145 | .091 | .040 |
| | 2,1 | .259 | .142 | .080 | .035 | .158 | .082 | .039 | .017 | .246 | .146 | .080 | .039 |
| | 5,1 | .251 | .149 | .084 | .038 | .163 | .083 | .046 | .016 | .254 | .152 | .095 | .049 |
| 10,20 | 1,1 | .254 | .142 | .078 | .037 | .169 | .082 | .037 | .014 | .247 | .147 | .082 | .040 |
| | 2,1 | .347 | .217 | .132 | .063 | .243 | .141 | .077 | .029 | .331 | .216 | .144 | .080 |
| | 5,1 | .436 | .304 | .213 | .125 | .313 | .194 | .124 | .063 | .408 | .288 | .202 | .127 |
| | 1,2 | .185 | .097 | .045 | .017 | .111 | .044 | .019 | .007 | .176 | .095 | .046 | .020 |
| | 1,5 | .120 | .050 | .025 | .007 | .064 | .023 | .006 | .002 | .127 | .059 | .025 | .011 |
| 20,20 | 1,1 | .256 | .150 | .078 | .032 | .163 | .075 | .040 | .012 | .247 | .149 | .084 | .040 |
| | 2,1 | .275 | .151 | .082 | .039 | .176 | .086 | .046 | .016 | .252 | .151 | .091 | .047 |
| | 5,1 | .245 | .140 | .076 | .035 | .176 | .085 | .043 | .016 | .266 | .159 | .092 | .048 |
| | | DD | | | | EE | | | | CD | | | |
| 10,10 | 1,1 | .251 | .151 | .088 | .039 | .310 | .189 | .117 | .064 | .262 | .149 | .077 | .037 |
| | 2,1 | .253 | .149 | .087 | .041 | .332 | .209 | .121 | .052 | .253 | .147 | .078 | .034 |
| | 5,1 | .244 | .147 | .093 | .051 | .323 | .211 | .126 | .062 | .255 | .155 | .094 | .049 |
| 10,20 | 1,1 | .249 | .143 | .084 | .041 | .303 | .181 | .109 | .054 | .253 | .146 | .086 | .040 |
| | 2,1 | .337 | .217 | .151 | .079 | .409 | .286 | .193 | .108 | .335 | .204 | .123 | .066 |
| | 5,1 | .419 | .299 | .214 | .131 | .512 | .383 | .279 | .178 | .422 | .292 | .205 | .124 |
| | 1,2 | .188 | .098 | .059 | .026 | .232 | .125 | .061 | .026 | .189 | .091 | .048 | .018 |
| | 1,5 | .128 | .059 | .029 | .010 | .172 | .081 | .039 | .013 | .129 | .061 | .028 | .011 |
| 20,20 | 1,1 | .255 | .147 | .093 | .048 | .319 | .191 | .115 | .063 | .266 | .152 | .082 | .037 |
| | 2,1 | .256 | .158 | .098 | .047 | .316 | .194 | .114 | .063 | .249 | .154 | .092 | .046 |
| | 5,1 | .265 | .155 | .090 | .048 | .320 | .198 | .123 | .061 | .262 | .150 | .091 | .043 |

Table 7--Continued

| GROUP SIZE 1 2 | VAR 1 2 | AB .10 | AB .05 | AB .025 | AB .01 | AC .10 | AC .05 | AC .025 | AC .01 | AE .10 | AE .05 | AE .025 | AE .01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10,10 | 1,1 | .194 | .098 | .051 | .019 | .249 | .144 | .081 | .040 | .286 | .173 | .098 | .043 |
| | 2,1 | .216 | .116 | .063 | .030 | .241 | .135 | .074 | .032 | .268 | .156 | .094 | .045 |
| | 5,1 | .254 | .147 | .088 | .043 | .258 | .148 | .091 | .049 | .262 | .147 | .086 | .040 |
| 10,20 | 1,1 | .236 | .133 | .075 | .035 | .257 | .141 | .084 | .035 | .274 | .153 | .087 | .043 |
| | 2,1 | .312 | .189 | .115 | .052 | .342 | .220 | .140 | .069 | .345 | .222 | .142 | .075 |
| | 5,1 | .432 | .294 | .201 | .124 | .439 | .301 | .213 | .135 | .450 | .324 | .224 | .139 |
| | 1,2 | .140 | .065 | .033 | .013 | .185 | .092 | .047 | .019 | .219 | .109 | .053 | .022 |
| | 1,5 | .085 | .029 | .009 | .001 | .135 | .063 | .026 | .011 | .159 | .073 | .030 | .011 |
| 20,20 | 1,1 | .226 | .122 | .062 | .026 | .255 | .148 | .085 | .042 | .289 | .167 | .095 | .044 |
| | 2,1 | .223 | .116 | .067 | .029 | .262 | .154 | .090 | .039 | .285 | .163 | .091 | .038 |
| | 5,1 | .247 | .129 | .078 | .033 | .266 | .158 | .087 | .038 | .274 | .160 | .093 | .045 |

| GROUP SIZE 1 2 | VAR 1 2 | BA .10 | BA .05 | BA .025 | BA .01 | CA .10 | CA .05 | CA .025 | CA .01 | EA .10 | EA .05 | EA .025 | EA .01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10,10 | 2,1 | .192 | .098 | .045 | .017 | .263 | .150 | .081 | .038 | .297 | .183 | .106 | .059 |
| | 5,1 | .178 | .087 | .043 | .016 | .250 | .149 | .091 | .045 | .307 | .186 | .114 | .060 |
| 10,20 | 1,1 | .195 | .094 | .049 | .020 | .249 | .133 | .073 | .037 | .293 | .181 | .106 | .049 |
| | 2,1 | .253 | .140 | .080 | .038 | .332 | .210 | .127 | .070 | .375 | .261 | .173 | .092 |
| | 5,1 | .345 | .218 | .134 | .073 | .426 | .301 | .210 | .139 | .484 | .353 | .259 | .161 |
| | 1,2 | .134 | .063 | .028 | .010 | .175 | .085 | .041 | .017 | .210 | .114 | .059 | .023 |
| | 1,5 | .102 | .043 | .016 | .006 | .126 | .056 | .022 | .007 | .136 | .060 | .026 | .009 |
| 20,20 | 2,1 | .192 | .105 | .049 | .017 | .252 | .145 | .082 | .038 | .297 | .178 | .097 | .047 |
| | 5,1 | .179 | .096 | .050 | .021 | .246 | .142 | .080 | .039 | .313 | .187 | .108 | .054 |

Table 7--Continued

| GROUP SIZE 1 2 | VAR 1 2 | ALPHA = .10 | .05 | .025 | .01 | .10 | .05 | .025 | .01 | .10 | .05 | .025 | .01 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | BC | | | | BE | | | | CE | | | |
| 10,10 | 1,1 | .196 | .111 | .058 | .022 | .227 | .125 | .066 | .025 | .290 | .170 | .099 | .047 |
| | 2,1 | .205 | .108 | .054 | .023 | .221 | .126 | .067 | .031 | .272 | .157 | .089 | .042 |
| | 5,1 | .203 | .102 | .052 | .025 | .201 | .105 | .052 | .020 | .269 | .155 | .091 | .049 |
| 10,20 | 1,1 | .197 | .097 | .046 | .014 | .229 | .125 | .064 | .025 | .278 | .164 | .095 | .051 |
| | 2,1 | .262 | .147 | .084 | .042 | .276 | .160 | .093 | .039 | .343 | .219 | .137 | .077 |
| | 5,1 | .326 | .213 | .127 | .062 | .356 | .233 | .154 | .085 | .421 | .301 | .213 | .134 |
| | 1,2 | .147 | .067 | .031 | .013 | .166 | .083 | .038 | .015 | .216 | .115 | .058 | .023 |
| | 1,5 | .110 | .047 | .021 | .006 | .147 | .070 | .034 | .013 | .155 | .073 | .031 | .011 |
| 20,20 | 1,1 | .205 | .110 | .054 | .023 | .233 | .136 | .075 | .034 | .294 | .171 | .094 | .044 |
| | 2,1 | .207 | .119 | .062 | .025 | .218 | .106 | .058 | .022 | .280 | .163 | .102 | .053 |
| | 5,1 | .194 | .100 | .054 | .023 | .219 | .117 | .062 | .022 | .272 | .163 | .101 | .051 |
| | | CB | | | | EB | | | | EC | | | |
| 10,10 | 2,1 | .213 | .114 | .061 | .022 | .271 | .159 | .086 | .036 | .292 | .175 | .103 | .046 |
| | 5,1 | .242 | .144 | .085 | .046 | .321 | .187 | .111 | .058 | .328 | .207 | .126 | .071 |
| 10,20 | 1,1 | .220 | .118 | .062 | .027 | .270 | .156 | .089 | .037 | .294 | .172 | .101 | .048 |
| | 2,1 | .321 | .197 | .125 | .067 | .394 | .264 | .170 | .094 | .401 | .271 | .175 | .099 |
| | 5,1 | .413 | .295 | .208 | .138 | .483 | .356 | .255 | .165 | .505 | .376 | .278 | .181 |
| | 1,2 | .147 | .074 | .035 | .015 | .172 | .082 | .039 | .014 | .212 | .112 | .060 | .026 |
| | 1,5 | .085 | .032 | .013 | .004 | .100 | .041 | .015 | .004 | .129 | .066 | .031 | .012 |
| 20,20 | 2,1 | .226 | .127 | .069 | .032 | .270 | .159 | .090 | .045 | .276 | .178 | .102 | .055 |
| | 5,1 | .247 | .133 | .074 | .037 | .302 | .185 | .118 | .063 | .320 | .212 | .131 | .067 |

Note: A, B, C, D, E refer to forms of heteroscedasticity.

## DISCUSSION

ANCOVA appears to be robust to the violation of the assumptions of homoscedasticity and homogeneity of variance both singly and in combination, when group sizes are equal. In every condition where group sizes and variances were heterogeneous the goodness-of-fit hypothesis was rejected. When the larger variance was combined with the larger group size, the empirical significance levels of ANCOVA were conservative, and when the larger variance was combined with the smaller group size, the empirical alpha levels were nonconservative. The effect of the violation of the assumption of homogeneity of variance on the empirical F distribution of ANCOVA parallels the results obtained by Norton (as reported in Lindquist, 1953) and Box (1954) when investigating the effect of heterogeneity of variance on the empirical F distribution of ANOVA. In addition, the results obtained by Hamilton (1972), McClaren (1972), and Peckham (1968), where they investigated the simultaneous violation of the assumptions of homogeneity of variance and regression, are replicated.

The failure to find any condition where heteroscedasticity alone was responsible for the rejection of the goodness-of-fit hypothesis leads one to the conclusion that the assumption of homoscedasticity is not important to ANCOVA. Thus, if variances are homogeneous, the transformation of heteroscedastic data before using ANCOVA, as was suggested by Elashoff (1969), may be unnecessary.

The Johnson-Neyman technique is robust to the violation of the assumption of homogeneity of variance when group sizes are equal. However, when group sizes are unequal heterogeneity of variance biases the probabilities of obtaining regions of significance in the same direction as found with ANCOVA. That is, the larger variance combined with the smaller group size produces a non-conservative bias and the larger variance combined with the larger group size produces a conservative bias.

The probability that a fixed X would be included in a region of significance was consistently determined by the form of heteroscedasticity. If the shapes of the probability distributions shown in Table 6 are compared to the shapes of heteroscedasticity in Figure 1, the following conclusions emerge:

1. When $\sigma^2_{y|x_{ij}}$ is constant across $X_{ij}$, the probability that $X_{ij}$ is included in a region of significance is constant and equal to the nominal alpha level.

2. When $\sigma^2_{y|x_{ij}}$ is greatest for the central value of $X_{ij}$ and smallest for the tails, the probability that $X_{ij}$ is included in a region of significance is greatest for the central $X_{ij}$ and smallest for the tails, and in general, the probabilities are conservative.

3. When the $\sigma^2_{y|x_{ij}}$ is greatest at either end value and progressively decreases to the opposite end value, the probability that $X_{ij}$ will be included in a region of significance follows the size of $\sigma^2_{y|x}$. The average significance level is close to the nominal level of significance.

4. When $\sigma^2_{y|x_{ij}}$ is smallest for the center $X_{ij}$ and largest at either end, the probability that $X_{ij}$ will be included in a region of significance is greatest at the end values of X and smallest for the central $X_{ij}$. In general, the empirical probabilities are nonconservative.

The effect of the form of heteroscedasticity when combined with a different form of heteroscedasticity is partially determined by the average variance of the group. The form of heteroscedasticity combined with the larger variance has a greater influence on the form of the probability distribution for fixed values of X.

Although the magnitude of the empirical values of the probabilities are further influenced by the condition of unequal group sizes and heterogeneity of variance, the effects of the form of heteroscedasticity outlined above hold constant. That is, the relative differences in the probabilities associated with each $X_{ij}$ is fixed by the form of heteroscedasticity, but the size of the probabilities are biased by the combination of unequal group sizes and heterogeneity of variance.

The probability of finding any region of significance greatly exceeded the nominal significance level for all forms of heteroscedasticity. As explicated by Potthoff (1964) the Johnson-Neyman technique may be used to specify the region of significance, but does not set simultaneous confidence bounds. For example, one can say with 95 percent confidence that for any specific point P within the region of significance that there is a true difference between the two groups when nominal alpha equals .05. One cannot say with 95 percent confidence that for all points within the region of significance there is a true difference in performance for the two groups.

Although not directly related to the research questions of the dissertation, it is of interest to inquire how to set a simultaneous confidence bound equal to the nominal alpha level. Potthoff suggests a method for setting simultaneous confidence bounds which is basically identical with the defining inequality for the Johnson-Neyman region of significance with the exception that $F_{1,n_1+n_2-4;\alpha}$ is replaced by $(r+1)F_{r+1,n_1+n_2-4;\alpha}$, where r is the number of concomitant variables. When this procedure was tested for the simulated experimental condition of equal group sizes (10,10) and equal variances ($\sigma^2_{y|x}=1, \sigma^2_{y|x}=1$) the probability of finding a region of significance matched the nominal $\alpha$ level as shown in Table 8.

Table 8

Empirical Results Obtained Using Potthoff's Simultaneous Confidence Bounds When All Assumptions Are Met and Group Size Equals 10

| | Nominal Alpha | | | |
|---|---|---|---|---|
| | .10 | .05 | .025 | .01 |
| Simultaneous Confidence Coefficient | .099 | .047 | .025 | .011 |

Therefore, it is recommended that Potthoff's procedure for obtaining simultaneous confidence bounds for the Johnson-Neyman technique be utilized when one wishes to obtain a region where one can state with 1-$\alpha$ confidence that there is a treatment effect simultaneously for all points within the region.

A second approach to maintaining the experiment-wise Type I error rate at the nominal significance level is to follow the procedure suggested by Johnson and Jackson (1959) and Abelson (1953). First, test for homogeneity of regression; if this hypothesis is rejected proceed with the Johnson-Neyman technique, otherwise, use ANCOVA.

The marked effect of heteroscedasticity on the Johnson-Neyman technique suggests that when heteroscedasticity is observed in the data that the use of the Johnson-Neyman technique is not appropriate if the heteroscedasticity cannot be eliminated or minimized. If forms C and D are present in the data it may be possible to apply a variance-stabilizing transformation (Dayton, 1970). The problem of estimating and testing regression coefficients when $\sigma^2_{y|x_{ij}}$ is a function of $X_{ij}$ has been discussed by Rutemiller and Bowers (1968) and Levenbach (1973). However, the proper way of dealing with heteroscedasticity was a problem presented in the original Johnson and Neyman article (1936) and it has not been solved to date.

## SUMMARY

The robustness of the Johnson-Neyman technique and analysis of covariance (ANCOVA) to violations of assumptions of homoscedasticity and homogeneity of variance was tested through use of Monte Carlo computer procedures. The study simulated a one-way, fixed-effects analysis with two treatment groups, one criterion, Y, and one covariate, X. Five fixed values of the covariate were selected with zero mean and unit variance, while the values of Y were varied randomly with a constant regression coefficient of .75. Four combinations of group sizes (10,10; 10,20;20,10;20,20), five combinations of group variances (1,1;1,2;2,1;1,5;5,1), and five forms of heteroscedasticity (combined in 18 different

pairs), were studied. These conditions were combined to produce 186 different simulated experimental conditions. For each simulated condition, 3000 pseudo-random samples were generated and sampling distributions relevant to the Johnson-Neyman technique and ANCOVA were compiled.

Results indicated that ANCOVA is robust to violations of assumptions of homoscedasticity and homogeneity of variance, both singly and in combination, when group sizes were equal. For cases of different group sizes and heterogeneous variances a predictable bias was observed. When the larger variance was combined with the larger group size the bias was conservative. When the pairings were reversed the bias was non-conservative. The Johnson-Neyman technique was sensitive to violation of the assumption of homoscedasticity for both equal and unequal group sizes. The effect of heteroscedasticity was to order the probability that any fixed value of X would be included in a region of significance in a sequence parallel to the form of heteroscedasticity. That is, in general, as the variance for a fixed value of X increased, the probability of including that value of the covariate in a region of significance increased. As observed with ANCOVA, the Johnson-Neyman technique was robust to heterogeneity of variance when group sizes were equal. However, when group sizes were not equal the empirical probabilities were biased in a non-conservative direction when the larger variance was combined with the smaller group size, and in a conservative direction when the larger variance and larger group size were combined.

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